Sample/Model questions

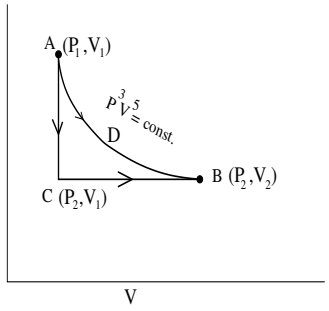
1. What are the forces acting on a particle described by the Lagrangian $L = \frac{1}{2}m(\dot{x}^2 - \omega^2 x^2)e^{\gamma t}$ (ω^2 and γ are positive constants)? Write down the Hamiltonian for this system.

2. A relativistic particle of mass m_0 and velocity $v = (4/5)c$ collides inelastically and fuses with an identical particle at rest. What are the velocity and the mass of the composite particle so formed ?

3. Estimate the energy of electrons that we need to use in an electron microscope to resolve a separation x.

4. By using Lagrange multipliers find the extrema of $f(x, y) = xy$ if (x, y) is restricted to the points on the ellipse $4x^2 + y^2 = 4$.

5. When you take a single breath, how many molecules of gas that you intake would have come from the dying breath of Julius Caesar? Ignore all absorptions, chemical reactions and treat the gas as a perfect gas.
(Assume: Radius of the earth $R = 6,400km$, Volume of gas in a single breath $\approx$ 1 litre, Thickness of the atmosphere $\approx 50km$, Mass of a proton $m_p \approx$ mass of a neutron $m_n = 1.7 \times 10^{-27}kg$, Density of the atmosphere on the surface of the earth $= 1.2kg/m^3$)

6. 

A gas is confined in a cylinder with a movable piston. It is observed that if the gas is adiabatically taken from state $A(P_1, V_1)$ to a state $B(P_2, V_2)$, in the PV diagram it follows a path ADB whose equation is $P^3V^5 =$ const.

If the gas is taken from state A to state B along a path ACB (as shown in figure), find the heat transfer to the system in the process ACB. (Assume all processes to be quasi-static).

7. (a) In finding the acceleration due to gravity g using a simple pendulum, the error in measurements of the length L of the pendulum is 1% and that of the time period T is also 1%. What is the error in g in %? Which measurement L or T needs more attention?

   (b) The measurement of length of any side of a cube gives $L = 1.2 \pm 0.1$cm. What is $(V \pm \Delta V)$ cm^3 where V is the volume of the cube?

8. Consider a periodic arrangement of atoms with coordinates given by $a(n_1\hat{i} + n_2\hat{j} + n_3\hat{k})$, where a is a constant length and n_1, n_2, n_3 are integers. +ve, -ve or zero. What would be the crystal structure if n_1, n_2 and n_3 satisfy the relation $n_1 + n_2 + n_3 =$ even integer? Give reasons for your answer.

9. In a double slit experiment illuminated by sodium vapour lamp having wave length 289 nm, the slits are 0.1 mm apart and the screen distance is 60 cm. Find the separation between the central peak and the first diffraction maximum on the screen.

10. For a particle in a two-dimensional box, $(0 < x < a)$, $(0 < y < b)$ find the probability of finding the particle within $0 < x < a/3$, if it's in the lowest energy state.